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Minority opinion spreading in random geometry

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Abstract. The dynamics of spreading of the minority opinion in public debates (a reform proposal, a behavior change, a military retaliation) is studied using a diffusion reaction model. People move by discrete step on a landscape of random geometry shaped by social life (offices, houses, bars, and restaurants). A perfect world is considered with no advantage to the minority. A one person-one argument principle is applied to determine locally individual mind changes. In case of equality, a collective doubt is evoked which in turn favors the Status Quo. Starting from a large in favor of the proposal initial majority, repeated random size local discussions are found to drive the majority reversal along the minority hostile view. Total opinion refusal is completed within few days. Recent national collective issues are revisited. The model may apply to rumor and fear propagation.

PACS. 89.75.Hc Networks and genealogical trees – 05.50.+q Lattice theory and statistics (Ising, Potts, etc.) – 87.23.Ge Dynamics of social systems

All over the world and more specifically in democratic countries public opinion seems to be rather conservative while facing a nationwide issue open to a public debate like for instance a reform proposal or a behavior change [1–3]. Even when the changes at stake are known to be desperately needed (medical evidences, danger of death, administrative inefficiencies) an initial hostile minority appears to be almost always able to turn the majority along its refusal position.

A symptomatic illustration of such a paradoxical social refusal was the year 2000 generalized failure of the French government to reform the academic system, the taxes collect and the agriculture system of economical help [3]. Another example is the Irish No to the Nice European treaty that came as a surprise to the Irish people itself [4]. Along with this reality some people could be tempted to consider reforms possible only using social violence or authoritarian top leadership decisions. It thus arises he fundamental question whether or not a reform can be decided democratically at least in principle.

To understand the reason of such a social inertia, most research has concentrated on analyzing the complicated psycho-sociological mechanisms involved in the process of opinion forming. In particular focusing on those by which a huge majority of people gives up to an initial minority view [1,2]. The main feature being that the prospect to loose definite advantages is much more energizing than the hypothetical gain of a reform. Such an approach is certainly realistic in view of the very active nature of minorities involved in a large spectrum of situations.

However in this letter we claim that in addition to the more aggressiveness and persuasive power of a threatened or very motivated minority there exists some basic and natural mechanism inherent to free pubic debate which makes the initial hostile minority to a full spreading over.

To ground our claim we present an extremely simple model to opinion forming using some concepts and techniques from the physics of disorder [5–8]. A diffusion reaction model is implemented on a landscape of random geometry. It does not aimed at an exact description of reality. But rather, by doing some crude approximations, it focuses on enlightening an essential feature of an otherwise very complex and multiple phenomena. In particular the holding of free public debate is shown to lead almost systematically to the total spreading of an initial hostile minority view within the initial proposal in favor huge majority. The associated dynamics of extreme public polarization at the advantage of the initial minority is found to result from the existence of asymmetric unstable thresholds [9,10] that are produced by the random occurrence of temporary local doubts. Some recent nation wide issues with respect to European construction are thus revisited [4,11]. The application to the phenomena of rumor and fear propagation is discussed [12].

We start from a population with N individuals, which have to decide whether or not to accept a reform proposal. At time t prior to the discussion the proposal has a support by $N_+(t)$ individuals leaving $N_-(t)$ persons against it. Each person is supposed to have an opinion making $N_+(t) + N_-(t) = N$. Associated individual probabilities

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to be in favor or against the proposal are thus,

$$P_{\pm}(t) \equiv \frac{N_{\pm}(t)}{N}, \quad (1)$$

with,

$$P_{+}(t) + P_{-}(t) = 1. \quad (2)$$

From this initial configuration, people start discussing the project. However they don't meet all the time and all together at once. Gatherings are shaped by the geometry of social life within physical spaces like offices, houses, bars and restaurants. This geometry determines the number of people, which meet at a given place. Usually it is of the order of just a few. Groups may be larger but it is much rare. Accordingly a given social life yields a random local geometry landscape characterized by a probability distribution for gathering sizes $\{a_i\}$ which satisfy the constraint,

$$\sum_{i=1}^{L} a_i = 1, \quad (3)$$

where $i = 1, 2, ..., L$ stands for respective sizes $1, ..., L$ with L being the larger group. While discussions can occur by chance, most are monitored through regular time break meetings like lunch, dinner, happy hour and late drink. At each encounter people meet with different people (friends, colleagues and acquaintances) at different areas of their social space. It extends from a building, a neighborhood, a town or another state. Thus, people gatherings occur in sequences in time, each one allowing a new local discussion. There, people may change their mind with respect to the reform proposal. During these meetings all individuals are assumed to be involved in one group gathering. It means a given person is, on average, taking part to a group of size i with probability a_i. The existence of one-person groups makes this assumption realistic. Each new cycle of multi-size discussions is marked by a time increment +1.

To emphasize the bare mechanism at work into the refusal dynamics which arises from local interactions we consider a perfect world. No advantage is given to the minority with neither lobbying nor organized strategy. Moreover an identical individual persuasive power is assumed for both sides. A one person – one argument principle is used to implement the psychological process of collective mind update. On this basis a local majority argument determines the outcome of the discussion. People align along the local initial majority view.

For instance a group of five persons with at start three in favor of the reform and two against it ends up with five people in favor of the reform. On the reverse two initial reform supporters leads to five persons against it. However in case of an even group this rule of one person – one argument leaves the local possibility of a temporary absence of a collective majority. The group is then at a tie within a non-decisional state. It doubts.

There we evoke a physical principle called the "inertia principle" which states that to put on motion a system, it is necessary to apply on it a force at least infinitesimally larger than the friction which holds on it to keep the system at rest. This principle can be put in parallel to the fundamental psychological asymmetry that exists between what is known and what is hypothetical. Therefore, to go along what is unknown, even if this unknown is supposed to be better, a local majority of at least one voice is necessary.

In terms of our model, at a tie the group does not move and thus decides not to move, *i.e.*, the full group turns against the reform proposal to preserve the existing situation. For instance a group of six persons with initially three in favor of the reform and three against it yields six persons against the reform. It is worth to stress that this is not an advantage given to the minority in terms of being more convincing. It is a collective outcome that results from a state of doubt. For instance dealing with a military retaliation people will avoid action unless there exist clear evidences for it. Accordingly having $P_{\pm}(t)$ at time t yields,

$$P_{+}(t+1) = \sum_{k=1}^{L} a_k \sum_{j=N[\frac{k}{2}+1]}^{k} C_j^k P_{+}(t)^j P_{-}(t)^{(k-j)}, \quad (4)$$

at time $(t+1)$ where $C_j^k \equiv \frac{k!}{(k-j)!j!}$ and $N[\frac{k}{2}+1] \equiv IntegerPart\ of\ (\frac{k}{2}+1)$. Simultaneously,

$$P_{-}(t+1) = \sum_{k=1}^{L} a_k \sum_{j=N[\frac{k}{2}]}^{k} C_j^k P_{-}(t)^j P_{+}(t)^{(k-j)}. \quad (5)$$

In the course of time, the same people will meet again randomly in the same cluster configuration. At each new encounter they discuss locally the issue at stake and change their mind according to above majority rule. To follow the time evolution of the reform support equation (3) is iterated until a stable value is reached. A monotonic flow is obtained towards either one of two stable fixed points $P_{+N} = 0$ and $P_{+Y} = 1$. The flow and its direction are produced by an unstable fixed point P_{+F} located in between P_{+N} and P_{+Y}. Its value depends on both the $\{a_i\}$ and L. We denote it the Faith point. For $P_{+}(t) < P_{+F}$ it exists a number n such that $P_{+}(t+n) = P_{+N} = 0$ while for $P_{+}(t) > P_{+F}$ it is another number m which yields $P_{+}(t+m) = P_{+Y} = 0$. Both n and m measure the required time at reaching a stable and final opinion. It is either a "Big Yes" to the reform at $P_{+Y} = 1$ or a "Big No" at $P_{+N} = 0$. Their respective values depend on the $\{a_i\}$, L and the initial value $P_{+}(t)$.

Repeated successive local discussions thus drive the whole population to a full polarization with a "Big Yes" to the reform project at $P_{+Y} = 1$, or a "Big No" at $P_{+N} = 0$. Accordingly, public opinion is not volatile. It stabilizes rather quickly (n and m are usually small numbers) to a clear stand.

Figure 1 shows the variation of $P_{+}(t+1)$ as function of $P_{+}(t)$ for two particular sets of the $\{a_i\}$. First one is $a_1 = a_2 = a_3 = a_4 = 0.2$ and $a_5 = a_6 = 0.1$ where $L = 6$. There $P_{+F} = 0.74$ which puts the required initial support to the reform success at a very high value of more than 74%. Simultaneously an initial minority above 26% is enough to

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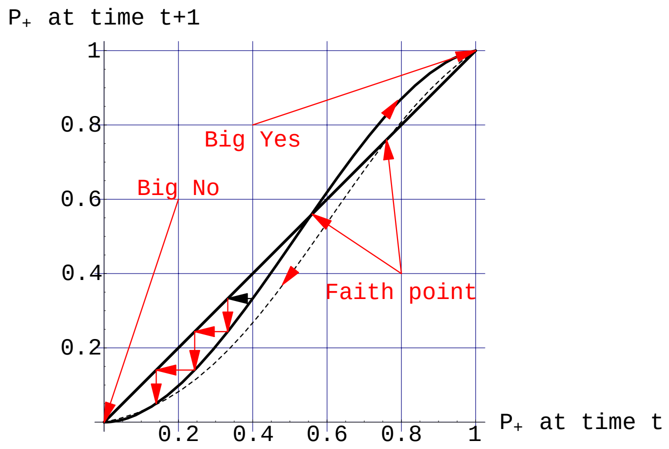

Fig. 1. Variation of $P_+(t+1)$ as function of $P_+(t)$. The dashed line is for the set $a_1 = a_2 = a_3 = a_4 = 0.2$, $a_5 = a_6 = 0.1$, $L = 6$ and $P_{+F} = 0.74$. The plain line is for the set $a_1 = 0$, $a_2 = 0.1$ and $a_3 = 0.9$ with $L = 3$ and $P_{+F} = 0.56$. Arrows show the direction of the flow.

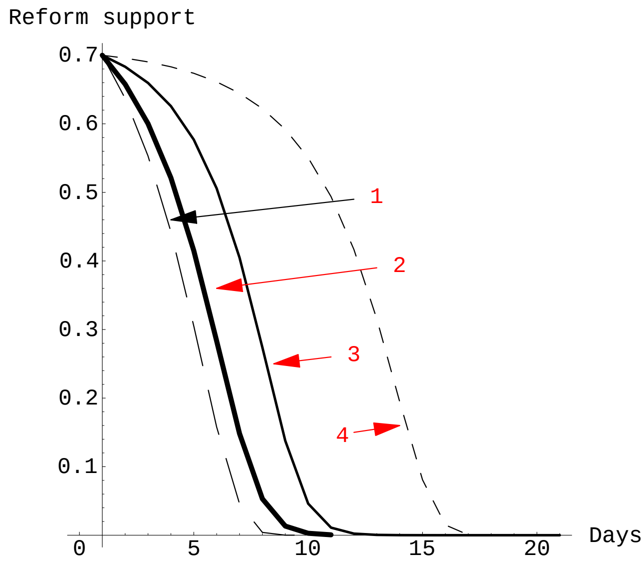

Fig. 2. Variation $P_+(t)$ as function of successive days with $L = 6$. The initial value at $t = 1$ is $P_+(1) = 0.70$. Long dashed line (1): $a_1 = 0$, $a_2 = \frac{1}{2}$, $a_3 = \frac{1}{2}$, $a_4 = a_5 = a_6 = 0$ with $P_{+F} = 1$. Heavy thick line (2): $a_1 = 0.2$, $a_2 = 0.3$, $a_3 = 0.2$, $a_4 = 0.2$, $a_5 = 0.1$ and $a_6 = 0$ with $P_{+F} = 0.85$. Other line (3): $a_1 = a_2 = a_3 = a_4 = 0.2$, $a_5 = a_6 = 0.1$. There $P_{+F} = 0.74$. Dashed line (4): $a_1 = 0$, $a_2 = 0.3$, $a_3 = 0.7$, $a_4 = a_5 = a_6 = 0$ with $P_{+F} = 0.71$.

produce a final total refusal. The second set is $a_1 = 0$, $a_2 = 0.1$ and $a_3 = 0.9$ with $L = 3$ and $P_{+F} = 0.56$. There the situation is much milder but also unrealistic since always pair discussions are much more numerous than just 10%.

To make a quantitative illustration of the dynamics refusal let us consider above first setting with an initial $P_+(t) = 0.70$ at time t. The associated series in time is $P_+(t+1) = 0.68$, $P_+(t+2) = 0.66$, $P_+(t+3) = 0.63$, $P_+(t+4) = 0.58$, $P_+(t+5) = 0.51$, $P_+(t+6) = 0.41$, $P_+(t+7) = 0.27$, $P_+(t+8) = 0.14$, $P_+(t+9) = 0.05$, $P_+(t+10) = 0.01$ and eventually $P_+(t+11) = 0.00$. Eleven cycles of discussion make all 70% of reform supporters to turn against it by merging with the initial 30% of reform opponents. On a basis of one discussion a day on average, less than two weeks is enough to a total crystallization of the No against the reform proposal. Moreover a majority against the reform is obtained already within six days (see Fig. 2).

Changing a bit the parameters with $a_1 = 0.2$, $a_2 = 0.3$, $a_3 = 0.2$, $a_4 = 0.2$, $a_5 = 0.1$ and $a_6 = 0$ gives $P_{+F} = 0.85$, a higher value, which makes any reform proposal quite impossible. How a realistic reform project could start with already more than 85% support in the population? Starting still from $P_+(t) = 0.70$ yields successively $P_+(t+1) = 0.66$, $P_+(t+2) = 0.60$, $P_+(t+3) = 0.52$, $P_+(t+4) = 0.41$, $P_+(t+5) = 0.28$, $P_+(t+6) = 0.15$, $P_+(t+7) = 0.05$, $P_+(t+8) = 0.01$ before $P_+(t+9) = 0.00$. The number of local meetings has shrieked from 11 to 9. Within ten days the whole population stands against the reform proposal. The initial 30% of opponents grow to more than fifty percent in less than four days (see Fig. 2).

It is the existence of an unstable fixed point between the two stable ones which produces the whole polarization dynamics. The stable ones are constant and independent of the a_i but the unstable one varies with both, sizes and the a_i distribution. To single out the specific contribution of each gathering size to the aggregation effect we now determine the associated unstable fixed point for groups

Table 1. Values of the various fixed points for each group size from two to six. $STP \equiv$ Stable fixed point and $UTP \equiv$ Unstable fixed point

Group Size	SFP Total No P_{+N}	UFP Faith Point P_{+F}	SFP Total Yes P_{+Y}
2	0	1	none
3	0	$\frac{1}{2}$	1
4	0	$\frac{1+\sqrt{13}}{6} \approx 0.77$	1
5	0	$\frac{1}{2}$	1
6	0	≈ 0.65	1

from two to six. Values are shown in Table 1. The flow landscape is identical for all odd sizes with an unstable fixed point at $\frac{1}{2}$. On the opposite for even sizes the unstable fixed point starts at one for size two and decreases to 0.65 at size six, *via* 0.77 at size four.

To illustrate the interplay dynamics between even and odd sizes, let us look at more details in the hypothetical case of discussion groups restricted to only two and three persons. Putting $a_1 = a_4 = .. = a_L = 0$, equation (4) reduces to,

$$P_+(t+1) = a_2 P_+(t)^2 + (1 - a_2)\{P_+(t)^3 + 3P_+(t)^2(1 - P_+(t))\}, \tag{6}$$

whose stable fixed points are still 0 and 1 with the unstable one located at,

$$P_{+F} = \frac{1}{2(1-a_2)} \cdot \qquad (7)$$

For $a_2 = 0$ (only three size groups) we recover $P_{+F} = \frac{1}{2}$ while it gets to $P_{+F} = 1$ already at $a_2 = \frac{1}{2}$. It shows the existence of pair discussion has a drastic effect on creating doubt that in turn produces a massive refusal spreading. Few days are now enough to get a total reform rejection.

Keeping an initial $P_+(t) = 0.70$ the time series become $P_+(t+1) = 0.64$, $P_+(t+2) = 0.55$, $P_+(t+3) = 0.44$, $P_+(t+4) = 0.30$, $P_+(t+5) = 0.16$, $P_+(t+6) = 0.05$ before $P_+(t+7) = 0.00$. The reform supporters falling down is extremely sharp as shown in Figure 2. Within a bit more than two days a majority of the people is already standing against the reform that yet started with a seventy- percent support. Seven days latter the proposal is completely out of any reach with not one single supporter. Considering instead only thirty percent of pair discussion groups, the falling is weakened but yet within sixteen days we have $P_+(t+16) = P_N 0.00$ (see Fig. 2).

Clearly an infinite number of combinations of the $\{a_i\}$ is possible. However the existence of these temporary local doubts which ultimately produces a strong polarization towards social refusal is always preserved. At this stage it is worth to stress that in real life situations not every person is open to a mind change. Some fractions of the population will keep on their opinion whatever happens. Including this effect in the model will not change qualitatively the results. It will make the polarization process not total with the two stable fixed points shifted towards respectively larger and smaller values than zero and one.

To give some real life illustrations of our model, we can cite events related to the European Union which all came as a surprise. From the beginning of its construction there have been never a large public debate in most of the involved countries. The whole process came trough government decisions tough most people always have seemed to agree on this construction. At the same time European opponents have been systematically urging for public debates. Such a demand sounds like absurd knowing a majority of people favor the European union. But anyhow most European governments have been reluctant to held referendum on the issue.

At odd, several years ago French president Mitterand decides to run a referendum to accept the Maastricht agreement [11]. While a large success of the Yes was given for granted it indeed made it just a bit beyond the required fifty percent. The more people were discussing, the less support there was for the proposal. It is even possible to conjuncture that an additional two weeks extension of the public debate would have make the No to win. The very recent Irish No [4] which came as a blow to all analysts may obey the same logic. The difference with the French case was certainly the weaker initial support. Of course addition political reasons to the No were also active.

To conclude, even tough our model is clearly very crude it does demonstrate the inherent polarization effect associated to the holding of democratic debates towards social immobility. Moreover this process was shown to be *de facto* anti-democratic since even in a perfect world it makes an initial minority refusal to almost systematically spread over in convincing very quickly the whole population. The existence of natural random local temporary doubts is instrumental in this phenomenon driven by the geometry of social life. As on how to remedy this reversal phenomenon, the first hint is to avoid the diffusion of the No *via* these localized temporary doubts. Direct and immediate votes would address this task preserving the democratic expression of an unbiased initial majority. But more thoughts and studies should be performed before getting to a clear proposal scheme. The model may generalize to a large spectrum of social, economical and political phenomena that involve propagation effects. In particular it could shed a new light on both processes of fear propagation and rumors spreading.

Rapid Note

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